Probability and statistics midterm exam (Model Two)
A- Probability

Question (1):

    A- An experiment consists of drawing three cards in succession from a well-shuffled ordinary deck of cards. Let A_1 be the event "king on first draw", A_2 the event "king on second draw" and A_3 the event "king on third draw." State in words the meaning of each of the following:

    i- $P(A_1 \cap A_2')$, ii- $P(A_1 \cup A_2)$, iii- $P(A_1' \cup A_2')$, iv- $P(A_1' \cap A_2' \cap A_3')$.

    B- If A is independent of B prove that (a) A is independent of B', (b) A' is independent of B'.

    C- Each of three identical jewelry boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in the other there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?

Question (2):

    A- The distribution function of a random variable X is given by F(x)
If P{X = 3} = 0 find (a) the constant c, (b) the density function, (c) P(X > 1), (d) P{1<X<2).

$$F(x) = \begin{cases} cx^3 & 0 \le x < 3 \\ 1 & x \ge 3 \\ 0 & x < 0 \end{cases}$$

    B- Let X and Y be continuous random variables having joint density function f(x,y) Determine (a) the constant c, b) P(X < ½ , Y > ½), (c) P(1/4 < X < 3/4), (d) P(Y< ½), (e) whether X and Y are independent.

$$f(x, y) = \begin{cases} c(x^2 + y^2) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Question (3):

    A- Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys, b) 5 girls, (c) either 2 or 3 boys? Assume equal probabilities for boys and girls.

    B- If the heights of 300 students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches, how many students have heights (a) greater than 72 inches, (b) less than or equal to 64 inches, (c) between 65 and 71 inches inclusive, (d) equal to 68 inches. Assume the measurements to be recorded to the nearest inch.

    C- If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs (a) 0, (b) 1, (c) 2, (d) 3, (e) 4, (f) 5 bulbs will be defective.

B- Statistics

Question (4):

Five hundred ball bearings have a mean weight of 5.02 ounces and a standard deviation of 0.30 ounces. Find the probability that a random sample of 100 ball bearings chosen from this group will have a combined weight of (a) between 496 and 500 ounces, (b) more than 510 ounces.

Question (5):

A sample of five measurements of the diameter of a sphere were recorded by a scientist as 6.33, 6.37, 6.36, 6.32 and 6.37 centimeters. Determine unbiased and efficient estimates of (a) the true mean, (b) the true variance.